LIMITS REVIEW

1. For "well-behaved" functions – Direct Substitution:

   Example: $\lim_{x\to -3}(3x+2)=(3(-3)+2)=-7$

2. For "Not-So-Well-Behaved" functions – Remember to AVOID $\frac{0}{0}$

   a. Try to simplify first:

   $$\lim_{x\to 0}\frac{[\frac{1}{x+4}-\frac{1}{4}]}{x}$$

   $$\lim_{x\to 0}\frac{[\frac{1}{x+4}-\frac{1}{4}]}{x}\bullet\frac{4(x+4)}{4(x+4)}$$

   $$\lim_{x\to 0}\frac{4-(x+4)}{4x(x+4)}$$

   $$\lim_{x\to 0}\frac{-x}{4x(x+4)}$$

   $$\lim_{x\to 0}\frac{-1}{4(x+4)}$$

   $$-\frac{1}{16}$$

   b. Try to factor and cancel:

   $$\lim_{x\to 2}\frac{2-x}{x^2-4}$$

   $$\lim_{x\to 2}\frac{2-x}{(x+2)(x-2)}$$

   $$\lim_{x\to 2}\frac{-1}{x+2}$$

   $$-\frac{1}{4}$$

   ***Recall:

   x = 2 is a removable discontinuity

   x = -2 is a non-removable discontinuity

   c. Try to rationalize the numerator:

   $$\lim_{x\to 3}\frac{(\sqrt{x+1}-2)}{x-3}$$

   $$\lim_{x\to 3}\frac{(\sqrt{x+1}-2)}{x-3}\bullet\frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$$

   $$\lim_{x\to 3}\frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

   $$\lim_{x\to 3}\frac{x-3}{(x-3)(\sqrt{x+1}+2)}$$

   $$\lim_{x\to 3}\frac{1}{(\sqrt{x+1}+2)}$$

   $$\frac{1}{4}$$

INFINITE LIMITS: $\lim_{x \to c} = \pm\infty$

This does not mean that the limit exists. But it does tell you how the limit fails to exist and denotes the unbounded behavior of the function.

****VERTICAL ASYMPTOTES****

$$\lim_{x \to c} \frac{f(x)}{g(x)} \Rightarrow \frac{f(c)}{g(c)} \Rightarrow \frac{\#}{0}$$

*This means there is a vertical asymptote at *x*=*c* and an infinite limit.

Example: Find vertical asymptotes: f(x) = $\frac{x^2+1}{x^2-1}$

Denominator = 0
$x^2 - 1 = 0$
$(x - 1)(x + 1) = 0$
$x = 1$ or $x = -1$
$\therefore$ There are vertical asymptotes at $x = 1$ and $x = -1$.

DEFINITION OF THE DERIVATIVE OF A FUNCTION:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

**Remember the derivative is slope or the rate of change.

LIMITS AT INFINITY: $\lim_{x \to \infty} \frac{c}{x^r} = 0$

Avoid $\frac{\infty}{\infty}$ by dividing through by the highest power of *x* in the denominator.

****HORIZONTAL ASYMPTOTES****

If $\lim_{x \to \pm\infty} f(x) = L$, $y = L$ is a horizontal asymptote.

L'HOPITAL'S RULE:

Only works for indeterminate forms: $\frac{0}{0}, \frac{\infty}{\infty}, \frac{\infty}{0}, \frac{0}{\infty}$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$